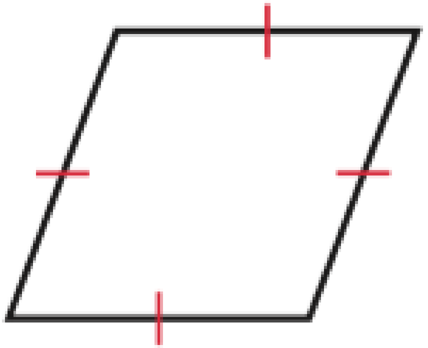
Chapter 6
Quadrilaterals

Section 4
Rhombuses, Rectangles, and Squares

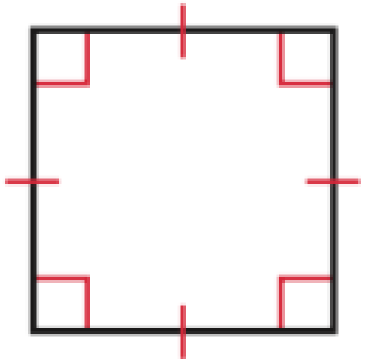
GOAL 1: Properties of Special Parallelograms

In this lesson you will study three special types of parallelograms: rhombuses, rectangles, and squares.



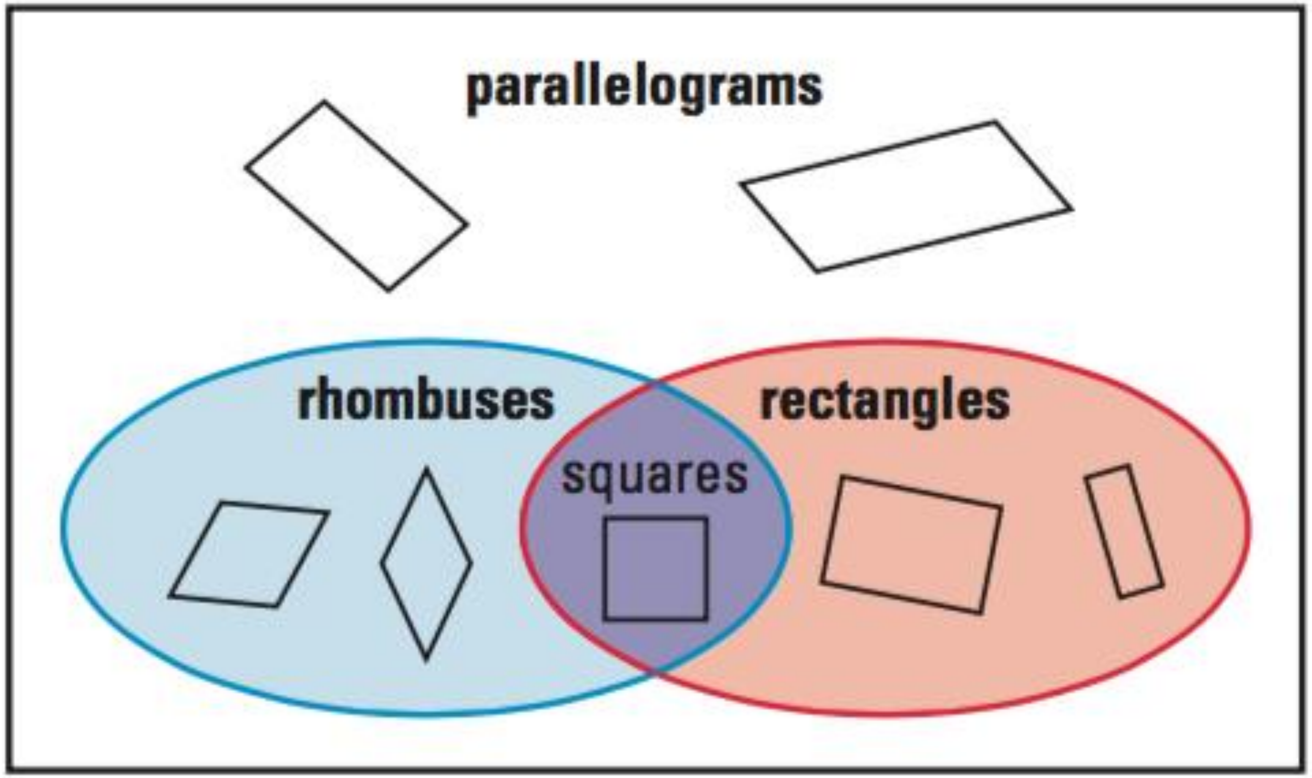
A **rhombus** is a parallelogram with four congruent sides.

A **rectangle** is a parallelogram with four right angles.



A **square** is a parallelogram with four congruent sides and four right angles.

The *Venn diagram* at the right shows the relationships among parallelograms, rhombuses, rectangles, and squares. Each shape has the properties of every group that it belongs to. For instance, a square is a rectangle, a rhombus, and a parallelogram, so it has all of the properties of each of those shapes.

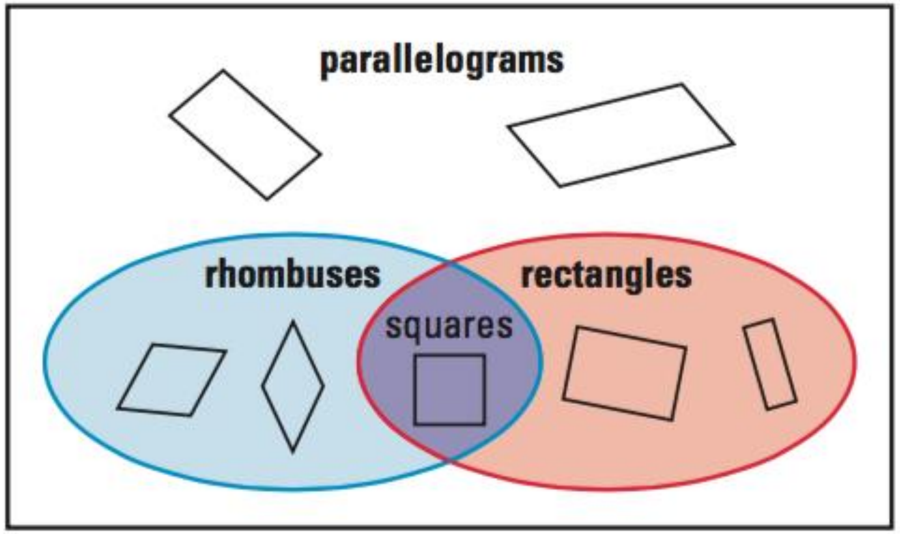


Example 1: Describing a Special Parallelogram

Decide whether the statement is always, sometimes, or never.

a) A rhombus is a rectangle

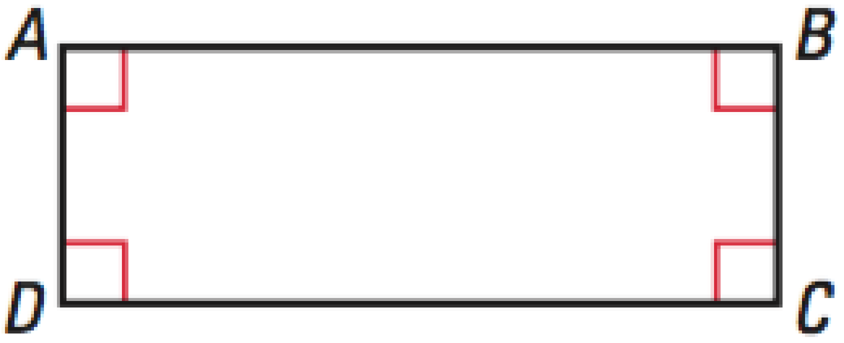
sometimes (square)



a) A parallelogram is a rectangle

sometimes

Example 2: Using Properties of Special Parallelograms



ABCD is a rectangle. What else do you know about ABCD?

- has 4 right angles
- has 2 sets of parallel lines
- opposite sides are congruent
- opposite angles are congruent
- consecutive angles are supplementary
- diagonals bisect

A rectangle is defined as a *parallelogram* with four right angles. But *any quadrilateral* with four right angles is a rectangle because any quadrilateral with four right angles is a parallelogram.

COROLLARIES ABOUT SPECIAL QUADRILATERALS

RHOMBUS COROLLARY

A quadrilateral is a rhombus if and only if it has four congruent sides.

RECTANGLE COROLLARY

A quadrilateral is a rectangle if and only if it has four right angles.

SQUARE COROLLARY

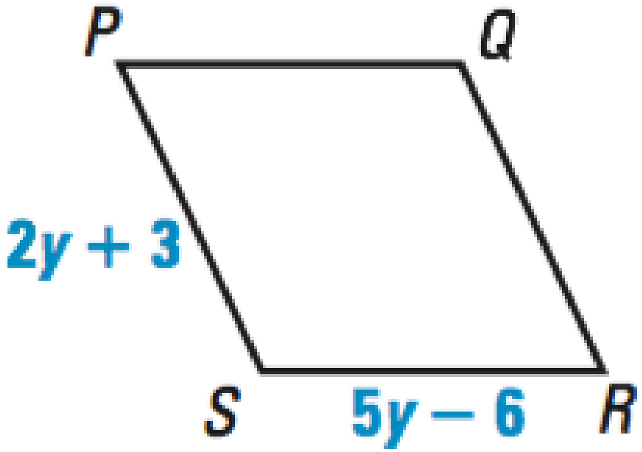
A quadrilateral is a square if and only if it is a rhombus and a rectangle.

You can use these corollaries to prove that a quadrilateral is a rhombus, rectangle, or square without proving first that the quadrilateral is a parallelogram.

Example 3: Using Properties of a Rhombus

In the diagram, PQRS is a rhombus. What is the value of y?

$$\cancel{2y} + 3 = 5y - 6$$
$$-\cancel{2y} \qquad -2y$$
$$3 = 3y - \cancel{6}$$
$$+6 \qquad +\cancel{6}$$
$$\frac{9}{3} = \frac{\cancel{3}y}{\cancel{3}}$$
$$3 = y$$



GOAL 2: Using Diagonals of Special Parallelograms

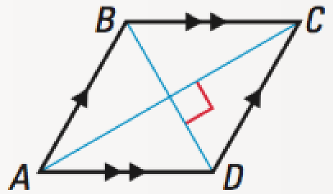
The following theorems are about diagonals of rhombuses and rectangles.

THEOREMS

THEOREM 6.11

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

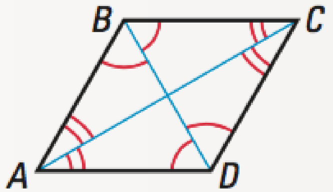
ABCD is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.



THEOREM 6.12

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

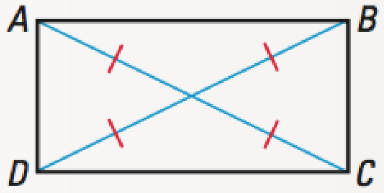
ABCD is a rhombus if and only if
$\overline{AC}$ bisects $\angle DAB$ and $\angle BCD$ and
$\overline{BD}$ bisects $\angle ADC$ and $\angle CBA$.



THEOREM 6.13

A parallelogram is a rectangle if and only if its diagonals are congruent.

ABCD is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.



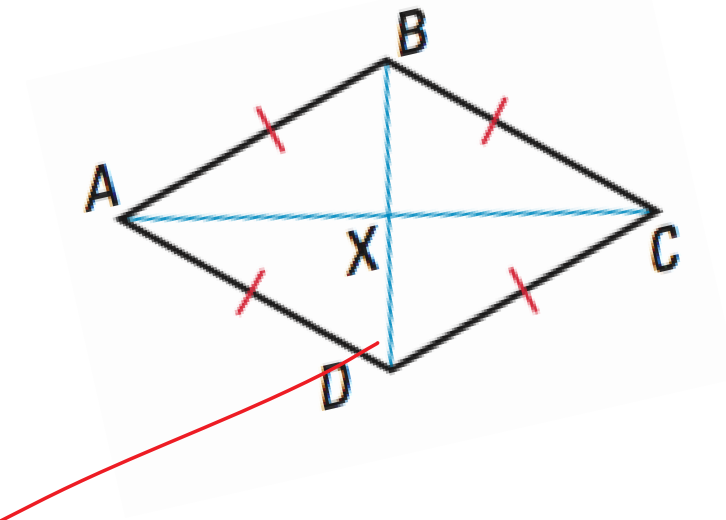
You can rewrite Theorem 6.11 as a conditional statement and its converse.

Conditional statement: If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

Converse: If a parallelogram is a rhombus, then its diagonals are perpendicular.

To prove the theorem, you must prove both statements.

Example 4: Proving Theorem 6.11



Given: ABCD is a rhombus.

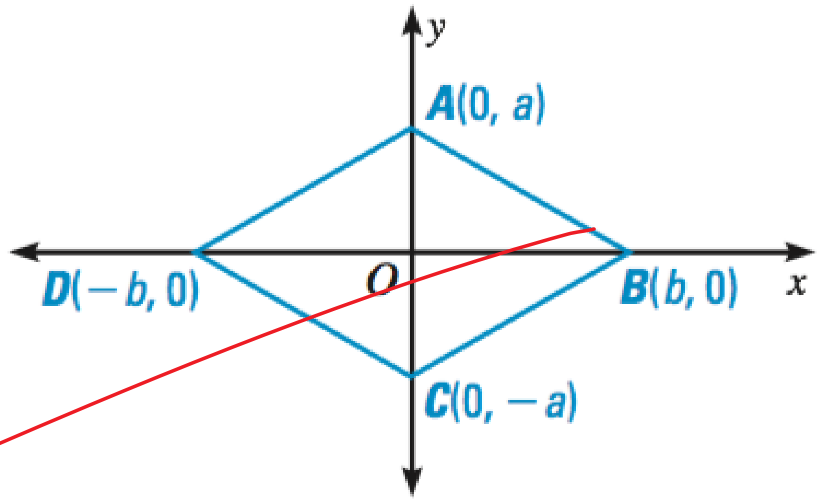
Prove: $AC \perp BD$

Statements	Reasons
1)	
2)	
3)	
4)	
5)	
6)	

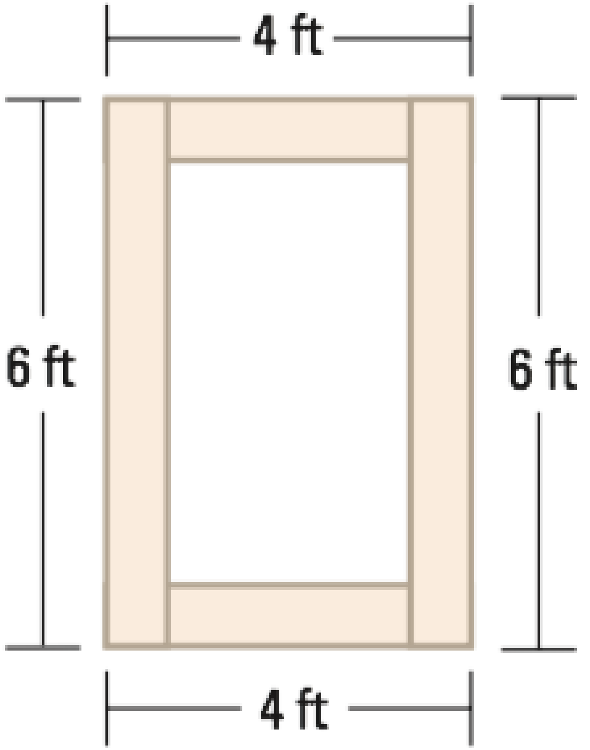
Example 5: Coordinate Proof of Theorem 6.11

Given: ABCD is a parallelogram, AC ⊥ BD

Prove: ABCD is a rhombus



Example 6: Checking a Rectangle



Carpentry: You are building a rectangular frame for a theater set.

a) First, you nail four pieces of wood together. What is the shape of the frame?

   parallelogram (don’t know if we have right angles)

a) To make sure the frame is a rectangle, you measure the diagonals. One is 7 feet 4 inches and the other is 7 feet 2 inches. Is the frame a rectangle? Explain.

   no, diagonals aren’t congruent

EXIT SLIP